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RESEARCH MEMORANDUM

A METHOD FOR SIMULATING THE ATMOSPHERIC ENTRY OF LONG-RANGE BALLISTIC MISSILES

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RESEARCH MEMORANDUM

A METHOD FOR SIMULATING THE ATMOSPHERIC ENTRY OF LONG-RANGE BALLISTIC MISSILES

By A. J. Eggers, Jr.

SUMMARY

It is demonstrated with the aid of similitude arguments that a model launched from a hypervelocity gun upstream through a special supersonic nozzle should experience aerodynamic heating and resulting thermal stresses like those encountered by a long-range ballistic missile entering the earth's atmosphere. This demonstration hinges on the requirements that model and missile be geometrically similar and made of the same material, and that they have the same flight speed and Reynolds number (based on conditions just outside the boundary layer) at corresponding points in their trajectories. The hypervelocity gun provides the model with the required initial speed, while the nozzle scales the atmosphere, in terms of density variation, to provide the model with required speeds and Reynolds numbers over its entire trajectory. Since both the motion and aerodynamic heating of a missile tend to be simulated in the model tests, this combination of hypervelocity gun and supersonic nozzle is termed an atmospheric entry simulator.

INTRODUCTION

The aerodynamic heating of a long-range ballistic missile entering the earth's atmosphere poses problems of so serious a magnitude that the success or failure of the missile may well hinge on their solution. The extent to which these problems are solved can be fully determined only by flight tests, but such tests are, per se, very difficult, time consuming, and costly. It is appropriate to inquire, therefore, if a method can be devised for simulating with relatively simple equipment on the ground, the aerodynamic heating and resulting thermal stresses in a ballistic missile. This question forms the starting point of the present paper which undertakes first to establish conditions of simulation and then to determine a practical method of simulation. In the course of events it will be discovered that simulation of heating goes hand in hand with simulation of motion of a missile; accordingly, the subject of this paper is, more generally, an atmospheric entry simulator.

SYMBOLS

A	reference area for drag evaluation, sq ft
$C_1(T)$	rate constant (see eq. (A1)), sec^{-1}
$C_2(T)$	rate constant (see eq. (A2)), cu ft/sec
C_D	drag coefficient, dimensionless
C_{f_l}	local skin-friction coefficient based on conditions just outside of boundary layer, dimensionless
C_f'	equivalent skin-friction coefficient, $\frac{1}{S}\int_S C_{f_l}\left(\frac{\rho_l}{\rho}\right)\left(\frac{V_l}{V}\right)dS$, dimensionless
D	typical model or missile dimension, ft
E	Young's modulus, lb/sq ft
H	heat transferred per unit area, $ft\text{-}lb/ft^2$
K	diffusivity, sq ft/sec
k	thermal conductivity, $ft\text{-}lb/sec\ ft^2(^{\circ}R/ft)$
L	length of nozzle, ft
M	flight Mach number, dimensionless
m	mass of missile or model, slugs
N	concentration of particles (i.e., number of particles per unit volume), ft^{-3}
n	distance normal to boundary, ft
$\bar{n}$	reduced normal distance, n/D, dimensionless
P_o	nozzle reservoir pressure, lb/sq ft
Q	total heat transferred, ft-lb
r	radius of curvature of body surface at stagnation point, ft

S	surface area, sq ft
T	temperature, $^{\circ}R$
T_o	nozzle reservoir temperature, $^{\circ}F$
t	time, sec
$\bar{t}$	reduced time, $\frac{t}{D^2}$, sec/sq ft
V	velocity, ft/sec
y'	altitude, ft
x,y,z	rectangular coordinates, ft (except when appearing as subscripts on stresses)
$\bar{x},\bar{y},\bar{z}$	reduced coordinates ($\bar{x} = x/D$, $\bar{y} = y/D$, $\bar{z} = z/D$), dimensionless
α	coefficient of thermal expansion, ft/ft-$^{\circ}R$
β	constant in density-altitude relation, ft^{-1}(see eq. (1))
δ	typical flow deflection angle, deg
θ	angle of flight path with respect to horizontal, deg
ν	Poisson's ratio, dimensionless
ρ	air density, slugs/cu ft
ρ_o	reference density, slugs/cu ft (reservoir density in nozzle)
$\sigma_x,\sigma_y,\sigma_z$	tensile or compressive stresses, lb/sq ft (see sketch, p. 9)
$\tau_{xy},\tau_{xz},\tau_{yz}$	shear stresses, lb/sq ft (see sketch, p. 9, and note $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, and $\tau_{yz} = \tau_{zy}$)
ξ,η,ζ	direction cosines of normal to a boundary, dimensionless

Subscripts

av	average conditions
e	conditions at "entrance" to earth's atmosphere or simulator
l	local conditions just outside of boundary layer

mi	missile
mo	model
s	stagnation conditions

ANALYSIS

In order to clarify the basic requirements of a simulator, this study is initiated with a review of some of the more important causes and effects of aerodynamic heating of long-range ballistic missiles. Following this review, the conditions of simulation will be set forth in a mathematical form designed to facilitate the choice of a practical simulator.

Motion and Heating of Long-Range Ballistic Missiles

It is a fundamental characteristic of ballistic missiles (see ref. 1) that speed begets range, with the result that hypervelocities in excess of 10,000 feet per second are required in order to obtain long ranges in excess of 1,000 miles. A long-range ballistic missile first attains hypervelocities near the end of powered flight. In this phase of flight and throughout the large majority of unpowered flight the vehicle should pass more or less unimpeded through the rarefied upper atmosphere of the earth, corresponding to altitudes in excess of several hundred thousand feet.[1] Its trajectory terminates, however, with a very rapid descent through all or part of the earth's relatively dense lower atmosphere. In this phase of flight, termed the atmospheric entry, retardation and severe aerodynamic heating of the missile can almost certainly be expected to occur (see ref. 2).

Retardation during atmospheric entry is caused by the combined action of pressure and viscous forces, while aerodynamic heating stems in the main from work done by viscous forces. In both cases it is aerodynamic rather than gravity forces which play the predominant role, with the result that motion and heating of the missile emerge as closely related phenomena. Thus changes in missile shape which affect motion will also affect heating. This fact can, as discussed in reference 2, be exploited in the design of missiles with reduced aerodynamic heating. The potential for excessive heating remains, however, as an unavoidable property of the long-range ballistic missile which enters the earth's atmosphere at hypervelocity.

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[1]It is presumed that the missile is, unlike the usual meteor, so large (say of the order of feet in typical dimension) that free molecule phenomena play a minor role in its motion and heating.

Excessive heating can have several effects. First and perhaps most serious of these effects is the development of high thermal stresses in the structure of the missile. These stresses tend, for example, to far overshadow the pressure induced stresses. In addition, there is the natural weakening of the missile material at high temperatures, so that structural failure may occur during atmospheric entry. There is, of course, the further possibility that intense aerodynamic heating will, as with meteors, cause burning and the ultimate destruction of the missile.

These, then, are some of the important causes and effects of aerodynamic heating of a long-range ballistic missile entering the atmosphere. They suggest that an atmospheric entry simulator might logically include means of simulating missile velocity, missile configuration (e.g., shape), and the lower portion of the earth's atmosphere. We will proceed from this suggestion to formulate our conditions of simulation.

Conditions of Simulation

The analysis to follow tacitly presumes the validity of many time-tested assumptions of aerodynamics, thermodynamics, and solid mechanics. In addition, however, it is predicated on the assumptions that during atmospheric entry (1) radiation has a secondary effect on missile heating, (2) gravity has a secondary effect on missile motion, (3) the flow-field freeze principle of Oswatitsch (ref. 3) holds for the missile, and (4) the thermal properties (e.g., specific heats) and transport properties (i.e., thermal conductivity and viscosity) of air are functions of temperature only. The first two assumptions are suggested by the calculations of references 1 and 2,[2] while the third assumption hinges essentially on the requirement that the square of the hypersonic similarity parameter for a missile be large compared to 1 during entry (i.e., $M^2\sin^2\delta >> 1$). For missiles of normal slenderness, the hyperspeed of entry tends to insure the satisfying of this requirement with the result that flight Mach number loses its significance as an important similarity parameter.[3] The last assumption is restrictive only in the event air tends to dissociate

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[2]The first assumption is, of course, best suited to "relatively light" missiles which are designed on the so-called "heat sink" principle, or, more generally, which are designed to maintain relatively cool surfaces.

[3]It is demonstrated in reference 3 that Mach number and stream angle in the disturbed flow are independent of flight Mach number, provided $M^2\sin^2\delta >> 1$, and provided the air behaves ideally. In the event nonideal behavior, like changes in the specific heats, occurs as a result of high temperatures in the disturbed flow, then flight velocity tends to replace flight Mach number as the important index of motion since it is through this velocity (via kinetic energy) that high disturbed air temperatures are created.

(or possibly ionize) and it will therefore be treated in a discussion of these phenomena later in the paper (see section on "Performance Limitations").

Now it is convenient in discussing similitude to imagine a model counterpart to the missile and a test chamber counterpart to the atmosphere. Furthermore it is permissible for our purposes to proceed from the simplified equations of reference 2 for the convective heating of ballistic missiles. Thus for an isothermal atmosphere (which closely approximates the earth's lower atmosphere, see ref. 2)

$$\frac{\rho}{\rho_o} = e^{-\beta y'} \tag{1}$$

and there follows:

(a) Heat absorbed per unit mass at altitude y'

$$\frac{Q}{m} = \frac{1}{4}\left(\frac{C_f{}'S}{C_D A}\right)\left(V_e{}^2 - V^2\right) \tag{2}$$

where

$$V^2 = V_e{}^2 e^{-\frac{C_D \rho_o A}{\beta m \sin\theta_e} e^{-\beta y'}} \tag{3}$$

(b) Average rate of heat transfer per unit area

$$\frac{dH_{av}}{dt} = \frac{C_f{}'\rho_o V_e{}^3}{4} e^{-\beta y'} e^{-\frac{3}{2}\frac{C_D \rho_o A}{\beta m \sin\theta_e} e^{-\beta y'}} \tag{4}$$

(c) Rate of heat transfer to stagnation point

$$\frac{dH_s}{dt} = 6.8\times10^{-6}\sqrt{\frac{\rho_o}{r}}\, V_e{}^3 e^{-\frac{\beta y'}{2}} e^{-\frac{3}{2}\frac{C_D \rho_o A}{\beta m \sin\theta_e} e^{-\beta y'}} \tag{5}$$

According to equation (1) the density of the air in the test chamber must vary exponentially with the distance corresponding to altitude in the atmosphere. The more general implication is, of course, that the test chamber must duplicate variations of ρ/ρ_o in the atmosphere, whatever they may be, although the absolute magnitudes of ρ and ρ_o may be quite different from those in the atmosphere. The static temperature of the air in the test chamber is, as in the case of the atmosphere, presumed to be small by comparison to missile recovery temperature (see ref. 2 in connection with this point as it relates to the derivation of eqs. (2) through (5)).

It will be stipulated now that model and missile be geometrically similar in structure and configuration, and made of the same material. Furthermore, the condition is imposed that the model enter the test chamber at the same speed and temperature as the missile enters the atmosphere. Finally, it is required that model and missile have the same Reynolds numbers (based on local conditions outside the boundary layer) at corresponding points $\beta y'$ in their trajectories. By corresponding points it is meant where the product $\beta y'$ is the same for model and missile. It should be recognized, of course, that, in general, β and y' will individually be grossly different for model and missile.

It follows from these requirements and equation (2) that the heat transfer per unit mass Q/m to model and missile will be the same at corresponding points $\beta y'$ provided V is the same, since C_f' and $C_f'S/C_DA$ are the same. But from equation (3) the velocity V will be the same at corresponding $\beta y'$ only if $C_D\rho_oA/\beta m \sin\theta_e$ is the same. If the subscript mo refers to model and mi to missile, then the last provision may be written

$$\frac{(\rho_oD)_{mi}}{(\rho_oD)_{mo}} = \left(\frac{D_{mi}}{D_{mo}}\right)^2 \frac{(y'/\sin\theta_e)_{mo}}{(y'/\sin\theta_e)_{mi}} \tag{6}$$

But model and missile Reynolds numbers, velocities, and disturbed air temperatures[4] are the same, hence

$$(\rho_oD)_{mi} = (\rho_oD)_{mo} \tag{7}$$

[4]This observation with regard to disturbed air temperatures is easily verified by considering flow near the surface of a missile with a stagnation point. Assuming for simplicity that air in the disturbed flow behaves ideally, we have at the stagnation point $T_s \approx V^2/2C_p$ since $M^2 >> 1$. Then the temperature of the air just outside the boundary layer is given by the expression $T_l = \frac{V^2}{2C_p}\left(1 + \frac{\gamma-1}{2}M_l^2\right)^{-1}$. Hence if V and M_l are the same for model and missile, then T_l and V_l are the same, independent of ambient air temperature. We are assured of equal M_l's by the freeze principle (see footnote 3[redacted]

and equation (6) may be written

$$\left(\frac{y_e'}{\sin \theta_e}\right)_{mo} = L = \left(\frac{y_e'}{\sin \theta_e}\right)_{mi} \left(\frac{D_{mo}}{D_{mi}}\right)^2 \tag{8}$$

This expression fixes the length L of the test chamber in terms of the portion of the atmosphere to be simulated therein and the ratio of model to missile size.[5] If equation (8) and the previously set forth requirements are satisfied, then model and missile should experience equal heat transfer per unit mass, and hence equal average temperature rise at corresponding points in their trajectories. These quantities are significant, of course, because they tend to determine whether a missile will melt or perhaps burn during flight.

The next question is how do the heat-transfer rates compare in the case of model and missile? It is easily deduced from equations (4) and (5) and the conditions for equal heat transfer per unit mass that

$$\left(\frac{dH_{av}}{dt}\right)_{mo} = \frac{D_{mi}}{D_{mo}} \left(\frac{dH_{av}}{dt}\right)_{mi} \tag{9}$$

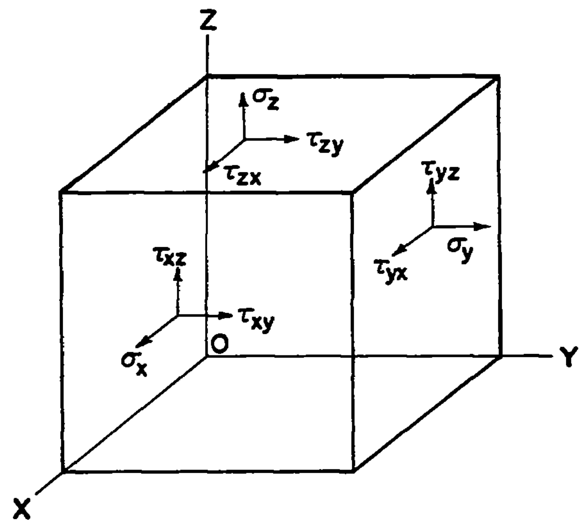
and

$$\left(\frac{dH_s}{dt}\right)_{mo} = \frac{D_{mi}}{D_{mo}} \left(\frac{dH_s}{dt}\right)_{mi} \tag{10}$$

at corresponding points $\beta y'$. That is to say, the average and stagnation point heat-transfer rates are higher for the model in proportion to the ratio of missile to model size. But perhaps the foremost importance of heat-transfer rates is, as discussed earlier, in how they influence thermal stresses in the missile structure and, for example, lead to ablation of surface material. Evidently, then, it would be most desirable if equations (9) and (10) implied equal thermal stresses in model and

[5]Since model and missile velocities are the same at corresponding points in their trajectories, it follows from equation (8) that the time of flight in the test chamber is reduced below that of atmospheric entry by the ratio $(D_{mo}/D_{mi})^2$.

missile. This possibility is easily checked using modified equilibrium thermal-stress equations for an unrestrained isotropic elastic body (see ref. (4) and sketch)



Stress notation

Thus

$$\left.\begin{aligned}
\frac{\partial\sigma_x}{\partial\bar{x}} + \frac{\partial\tau_{xy}}{\partial\bar{y}} + \frac{\partial\tau_{xz}}{\partial\bar{z}} - \frac{\alpha E}{1-2\nu}\frac{\partial T}{\partial\bar{x}} &= 0 \\
\frac{\partial\sigma_y}{\partial\bar{y}} + \frac{\partial\tau_{xy}}{\partial\bar{x}} + \frac{\partial\tau_{yz}}{\partial\bar{z}} - \frac{\alpha E}{1-2\nu}\frac{\partial T}{\partial\bar{y}} &= 0 \\
\frac{\partial\sigma_z}{\partial\bar{z}} + \frac{\partial\tau_{xz}}{\partial\bar{x}} + \frac{\partial\tau_{yz}}{\partial\bar{y}} - \frac{\alpha E}{1-2\nu}\frac{\partial T}{\partial\bar{z}} &= 0
\end{aligned}\right\} \tag{11}$$

where the stresses are not true thermal stresses, but rather they are the stresses produced by the "body forces," $\frac{-\alpha E}{1-2\nu}\frac{\partial T}{\partial\bar{x}}$, etc.

The corresponding compatibility conditions are

$$\left.\begin{aligned}
\nabla^2\sigma_x + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{x}^2} &= \frac{\alpha E}{1-2\nu}\left(\frac{\nu}{1-\nu}\nabla^2 T + 2\frac{\partial^2 T}{\partial\bar{x}^2}\right)\\
\nabla^2\sigma_y + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{y}^2} &= \frac{\alpha E}{1-2\nu}\left(\frac{\nu}{1-\nu}\nabla^2 T + 2\frac{\partial^2 T}{\partial\bar{y}^2}\right)\\
\nabla^2\sigma_z + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{z}^2} &= \frac{\alpha E}{1-2\nu}\left(\frac{\nu}{1-\nu}\nabla^2 T + 2\frac{\partial^2 T}{\partial\bar{z}^2}\right)\\
\nabla^2\tau_{yz} + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{y}\partial\bar{z}} &= 2\frac{\alpha E}{1-2\nu}\frac{\partial^2 T}{\partial\bar{y}\partial\bar{z}}\\
\nabla^2\tau_{xz} + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{x}\partial\bar{z}} &= 2\frac{\alpha E}{1-2\nu}\frac{\partial^2 T}{\partial\bar{x}\partial\bar{z}}\\
\nabla^2\tau_{xy} + \frac{1}{1+\nu}\frac{\partial^2\psi}{\partial\bar{x}\partial\bar{y}} &= 2\frac{\alpha E}{1-2\nu}\frac{\partial^2 T}{\partial\bar{x}\partial\bar{y}}
\end{aligned}\right\} \tag{12a}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial\bar{x}^2} + \frac{\partial^2}{\partial\bar{y}^2} + \frac{\partial^2}{\partial\bar{z}^2}$$

$$\psi = \sigma_x + \sigma_y + \sigma_z$$

and the boundary conditions are

$$\left.\begin{aligned}
\sigma_x\xi + \tau_{xy}\eta + \tau_{xz}\zeta &= \frac{\alpha E T}{1-2\nu}\xi\\
\sigma_y\eta + \tau_{yz}\zeta + \tau_{xy}\xi &= \frac{\alpha E T}{1-2\nu}\eta\\
\sigma_z\zeta + \tau_{xz}\xi + \tau_{yz}\eta &= \frac{\alpha E T}{1-2\nu}\zeta
\end{aligned}\right\} \tag{12b}$$

The heat-flow equation is (see ref. (5))

$$\nabla^2 T = \frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\partial^2 T}{\partial \bar{z}^2} = \frac{1}{K} \frac{\partial T}{\partial \bar{t}} \tag{13}$$

with boundary conditions

$$\left.\begin{aligned} &(T_{wall})_{initial} = \text{known} \\ &k \left.\frac{\partial T}{\partial \bar{n}}\right|_{\substack{\text{outer}\\ \text{surface}}} = D \frac{dH}{dt} \\ &\text{and (probably)} \\ &k \left.\frac{\partial T}{\partial \bar{n}}\right|_{\substack{\text{inner}\\ \text{surface}}} = 0 \end{aligned}\right\} \tag{14}$$

Now the thermal stresses are given by superposing the "hydrostatic" pressure $\alpha ET/1\text{-}2\nu$ on solutions to equations (11) consistent with the compatibility and boundary conditions (eqs. (12)) and the solution to the heat-flow equation (13) along with its boundary conditions (eqs. 14). But if equations (9) and (10) along with the requirements necessary for their development are satisfied, then equations (11) through (14) (and the "hydrostatic" pressure) are mathematically identical for model and missile. Hence, the thermal stresses must be the same in model and missile at corresponding $\beta y'$ in their trajectories.

Necessary conditions for simulating aerodynamic heating and resulting thermal stresses in a ballistic missile are then, according to this analysis, as follows. First, model and missile must be geometrically similar and made of the same material. In addition, they must have the same flight speeds and Reynolds numbers (based on local conditions outside the boundary layer) at corresponding points in their trajectories. Finally, in order to meet these conditions and to insure equal heating of model and missile, the test chamber must contain air at relatively low temperature and with variations in ρ/ρ_o along the model flight path equal to those in the atmosphere along the missile flight path.

With this knowledge we are in a position to consider the practical problems of simulating atmospheric entry.

A PRACTICAL ATMOSPHERIC ENTRY SIMULATOR

It is appropriate to determine first how to provide a model with the correct initial hypervelocity required for simulation. For this purpose it is suggested that a hypervelocity gun can be employed. A. C. Charters and his group at Ames Laboratory (see ref. 6) have, for example, launched a 22-caliber model from a hypervelocity (light-gas) gun at speeds up to 15,400 feet per second. This speed is 60 percent of satellite speed and corresponds to that of a ballistic missile (see ref. 1) with up to 2,000 miles range. Substantially higher speeds (corresponding to longer ranges) appear to be realizable with light-gas guns. Both speed and range fall, then, into the categories of interest in this paper.

The next question is how to provide a model test chamber which simulates the lower portion of the earth's atmosphere. For this purpose it is suggested that a special supersonic nozzle can be used to advantage. To illustrate, it was found in reference 2 (see fig. 4 therein) that the major part of the aerodynamic heating of a ballistic missile entering the atmosphere occurs over a range of about 100,000 feet in altitude. In this altitude range ρ/ρ_0 varies by a factor of about 10^{-2}. A corresponding variation in density can be obtained between the settling chamber and exit section of a Mach number 5 supersonic nozzle (see ref. 7). Imagine, then, a hypervelocity gun positioned to launch a model upstream along the axis of such a nozzle. The nozzle is designed to provide an essentially exponential variation in density along its axis, the density decreasing from a very high value in the settling chamber to a very low value at the exit. Accordingly, a model proceeding upstream through the nozzle encounters an increasingly dense atmosphere like that presented by the earth to a descending ballistic missile. Now, to be sure, unlike atmospheric air, the air in the nozzle is in motion. However, the air velocity is small by comparison to the hypervelocity of the model, and therefore this difference between nozzle air and atmospheric air should mar only slightly the function of the nozzle as a test chamber.[6]

The combination of hypervelocity gun and supersonic nozzle merits attention, then, as an atmospheric entry simulator. An example simulator of this type will therefore be considered next.

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[6]It is interesting to note that the supersonic nozzle when used in the proposed manner has compensating features; namely, as model velocity decreases, the air velocity decreases and, as model recovery temperature decreases, the air static temperature rises up toward atmospheric air temperature. These features are, of course, favorable to simulating atmospheric entry in that they tend to preserve the required similarity between atmospheric air and nozzle air. In this manner, too, the possibility of correcting for small differences between these media is enhanced.

Example Simulator

Consistent with the previous discussion, a Mach number 5 nozzle is chosen to simulate the earth's atmosphere over 100,000 feet of altitude. Now let us assume that ballistic missiles with up to 4,000 miles range are to be studied with this simulator. The corresponding range of atmospheric entrance angles θ_e is from 45° down to about 30° (see ref. 1). The length of the nozzle is fixed according to equation (8) by the maximum values of $(y_e'/\sin\theta_e)_{mi}$ and D_{mo}/D_{mi}. This value of $(y_e'/\sin\theta_e)_{mi}$ is, from the above specifications, $\frac{100,000}{\sin 30^\circ} = 200,000$ feet. The maximum value of D_{mo}/D_{mi} will be dictated by the size of the largest model which can be launched by the special gun available and by the size of the smallest missile to be simulated. For the purpose of this discussion it suffices to observe that the model size will probably be of the order of a fraction of an inch, while the missile size will probably be of the order of several feet. It follows that the maximum value of D_{mo}/D_{mi} should be of the order of 10^{-2}. In this event we have from equation (8) that the length of the nozzle is of the order of 20 feet, and from equation (7) the nozzle stagnation density is of the order of 100 times sea level air density. A missile 3 feet in diameter and weighing 5,000 pounds would thus be simulated by a model 0.36 inch in diameter and weighing 0.005 pound.

On the basis of these considerations our example simulator might appear something like the one shown in figure 1. The required nozzle stagnation densities are obtained for running times of the order of a second by using a settling chamber charged with high pressure air. A wide range of settling-chamber pressures and, hence, densities makes it possible to vary the range of altitudes simulated. The storage heater in the settling chamber maintains the air temperatures above those for which liquefaction can occur in the nozzle. The nozzle contour will tend to have very small slopes in the streamwise direction. Hence it is anticipated that flow in the nozzle will not depart radically from the one-dimensional type. Air from the nozzle passes into a vacuum tank which is of sufficient size to maintain the nozzle compression ratios required for supersonic flow during the course of a test. Observation windows are located at short intervals along the nozzle side walls to permit photographing models in flight and determining effects of aerodynamic heating. In this regard it would quite likely prove desirable to employ a spectrograph (or spectrographs) to identify the sources of radiation energy emitted in the vicinity of models. The simulator would be instrumented for measuring settling-chamber pressure and temperature and, of course, the time-distance history of models. Models would be launched from a hypervelocity gun located at or near the end of the nozzle, and they would be arrested in a catcher located in the settling chamber.

We have, then, some idea of how the proposed simulator might be employed in practice. It is important, however, to be aware of the points of difficulty which may limit the performance of the device as a simulator.

Performance Limitations

First, it should be noted that pressure-induced stresses in the model will be higher than those in the missile by the ratio D_{mi}/D_{mo}. This point may prove troublesome, although not unduly so if these stresses are very low (as they tend to be) in the missile. It should be remembered too that by proper model design (e.g., the use of internal pressurization or other bracing) this problem can be minimized. Careful attention must, of course, be given to both model and sabot design from the standpoint of minimizing stresses in the model during launching.

Another point of difficulty may be encountered if material properties (e.g., yield point) are a significant function of time under rapid heating conditions. There is some indication (see ref. 8 and papers cited therein) that at the very high heating rates of long-range ballistic missiles (corresponding to temperature-rise rates of the order of 100° F/sec and more), the importance of time is small. It is indicated too that this remains true, and, more important, that material properties remain essentially the same at the much higher heating rates produced in the simulator.[7] Far more information is needed, however, before the significance of time in the sense of this discussion can be fully assessed.

A further question which should be raised is whether or not a body can distort fast enough with changes in temperature to remain in stress-strain equilibrium (i.e., acceleration terms negligible) as was assumed in deriving equations (11) and (12). This situation would be more serious in the case of the model than in the case of the missile. A crude check on the matter is easily obtained for a body which behaves elastically. To illustrate, the time required for the model to make small adjustments in stress-strain equilibrium should be of the same order as the time required for an elastic wave to travel the length of the model. For a typical steel model ($D = 1/3$ in.) in the simulator, this latter time would be about 10^{-6} seconds. If we assume that the model experiences a total temperature rise of 1500° F while traveling through the simulator, then 10^{-6} seconds is also the time required for only about 1° temperature change in the model material. Accordingly, only small adjustments in stress-strain equilibrium are evidently required in this time, and hence

[7]This discussion should not be construed to mean that material properties are the same under conditions of high rates of heating as they are under steady state conditions (see ref. 8).

equilibrium should tend to be realized in model as well as missile. If time-dependent plastic deformation should become significant, then this statement obviously no longer holds.[8] Thus, for example, the simulator may not (in view of its foreshortened time scale) duplicate more than qualitatively a fracture process (see ref. 9) although it should tend to duplicate thermal deformation up to and including the beginning of fracture.

There is, too, the possibility of a missile being aerodynamically heated to temperatures where it will burn during descent through the earth's atmosphere. The simulator should tend to duplicate conditions leading up to this phenomenon; however, there is a question as to how well burning would be duplicated. The complexity and lack of complete understanding of metal burning (see, e.g., ref. 10) preclude the possibility of obtaining a quantitative answer to this question at the present time. From the qualitative viewpoint it is reasonable to expect that the effects of increased partial pressure of oxygen (acting to increase burning rate) and the reduced model size (acting to decrease the amount of material to be burned) should combine in the simulator to compensate for its foreshortened time scale, thereby more nearly providing simulation of missile burning. Burning should be accurately simulated for cases where burning rate is directly proportional to the partial pressure of oxygen.

As a final point, it is appropriate to consider the matter of dissociation and association of the oxygen and nitrogen in air. The simulator produces essentially the same disturbed air temperatures as the missile entering the atmosphere. Accordingly, the potential for dissociation and association is duplicated by the simulator. At the present time, however, it is felt that these two phenomena obey different rate laws (see Appendix). In this event, neither the phenomena nor their effects on heating of a ballistic missile can be duplicated except by 1 to 1 simulation (i.e., the equivalent of flight tests with the full-scale missile). An indication of the possible error that this situation may introduce in tests with the proposed simulator can be obtained from the calculations of reference 11 which suggest that the net effect of equilibrium dissociation and association in free flight may be to increase the rate of heat transfer to a stagnation point by only about 50 percent.[9] This increase should be well within the design safety factor of a missile.

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[8] Thus with a slow process, like creep, model deformation would not simulate missile deformation (see, e.g., ref. 9), but rather, it would be less by the ratio of $(D_{mo}/D_{mi})^2$ due to the foreshortened time scale in the simulator. Even in the case of the missile, however, the entry time is so short (of the order of seconds) it seems unlikely that creep could play a major role in deforming the vehicle.

[9] Dissociation and association may have especially marked effects on stagnation point heat transfer since these phenomena can strongly affect both the inviscid and viscous flows in this region.

CONCLUDING REMARKS

It has been found that an atmospheric entry simulator consisting of a hypervelocity gun combined with a special supersonic nozzle may be used to study the conditions of extreme heat transfer and thermal stress which introduce such serious problems in the flight of a long-range ballistic missile. The effects of aerodynamic heating on the model can be observed with relative ease, and further, the tests can be conducted at a cost which is negligible by comparison to that of flight tests. Indeed, in the simplest test, the simulator could provide with one photograph of a model rather substantial evidence as to whether or not the corresponding missile would remain essentially intact while traversing the atmosphere.

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National Advisory Committee for Aeronautics
Moffett Field, Calif., Sept. 15, 1955

APPENDIX A

DISSOCIATION AND ASSOCIATION RATES

The dissociation of O_2 and N_2 is thought (see, e.g., refs. 12 and 13) to obey the linear law

$$\frac{dN}{dt} = C_1(T)N \tag{A1}$$

where N is the concentration (or density in particles per unit volume) and $C_1(T)$ is the rate constant which usually depends only upon temperature. On the other hand, the corresponding association process (e.g., $O + O \rightarrow O_2$) is thought to obey the second-order law[1]

$$\frac{dN}{dt} = C_2(T)N^2 \tag{A2}$$

Now disturbed air velocities are the same in the case of model and missile, while disturbed air densities are higher in the case of the model by the ratio of D_{mi}/D_{mo}. It follows then that the simulator will tend to duplicate a rate process in which the percentage rate of change of concentration of a given type of particle is proportional to the concentration of that particle, namely,

$$\frac{1}{N}\frac{dN}{dt} = C(T)N \tag{A3}$$

But the simulator tends to duplicate temperature and type of particle; hence, it should duplicate the "association" rate process given by equation (A2) since $C(T) = C_2(T)$. It will apparently not, however, duplicate the "dissociation" rate process given by equation (A1).

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[1]Actually there is some question as to the correctness of either equation (A1) or (A2) (see e.g., ref. (14)) for pure media and there is the further complication of impurities which could lead, for example, to a third-order rate process.

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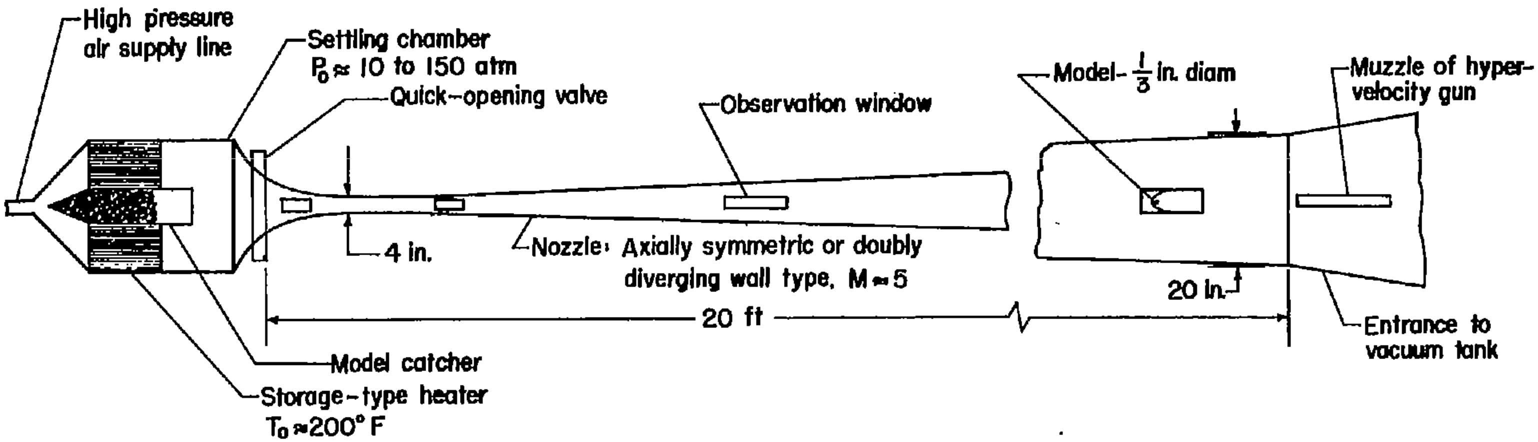

Figure 1.- Example atmospheric entry simulator.

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